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Lattice Boltzmann Method for Shallow Water Flows with Wet –Dry Interface

Weerasak Dee-am[1] Khamron Mekchay[2] Montri Maleewong[3]

Abstract

This study concerns the lattice Boltzmann method (LBM) for shallow water flow of dam-break problems, especially in wet-dry transitions where water flow from a wet area to a dry area. This problem of wetting-drying interface is known in literature that can cause difficulty for a standard LBM. In this study, the Taylor expansion and Chapman-Enskog procedure are considered to handle the problem without any spurious assumption, (Liu & Zhou, 2014). Moreover, the bed slope and the bed friction are also included in the scheme. The benchmark problems are also presented to confirm the study of the wet-dry problems.

Keywords: Lattice Boltzmann Method, Shallow water equations, Dam-break

1 Student from Applied Mathematics and Computational Science. Department of Mathematics and Computer Science. Faculty of Science. Chulalongkorn University. uueerasak@gmail.com

2 Assistant Professor. Department of Mathematics and Computer Science. Faculty of Science. Chulalongkorn University. khamron@chula.ac.th

3 Associate Professor. Department of Mathematics. Faculty of Science. Kasetsert University. montri@ku.ac.th

Introduction

Long-wave phenomena are commonly exist in nature such as wave run-up, Tsunami and solute transport in blood vessel, etc. One way to model these phenomena is to use the so called shallow water equations (SWE), which is powerful and efficient to simulate long-wave flow phenomena. Althrough according to much research have demonstrated the solving of the problem described by SWE using traditional numerical methods – such as the finite different method (FDM), the finite element method (FEM) and the finite volume method (FVM) – to simulate those problems with complex topography. These seem to be difficult to manage their codes and can cause many mistakes.

The new numerical method called lattice Boltzmann method (LBM) was developed and introduced in recent decades based on kenitic model (Succi, 2001). LBM is the mesoscopic method with simple arithmetic of just one parameter, the distribution function which is the function that describes the fluids particles. Lattice Boltzmann equation is the key equation of this method consisting of two crucial steps, collision step and the streaming step. In this equation the distribution function of the previous time is needed to calculate the distribution function of the present time. The connection between the distribution function and the unknown variables like the water velocity and the water depth is efficient. These give a better way to manage the code to calculate them all to save computer resources.

There are many results involved the LBM for shallow water flows. Rakwongwan & Maleewong (2013) solved the dam – break problem described by shallow water equations using LBM without the source terms such as bed slope and bed friction. Zhou (2002) presented the simulation of LBM for shallow water flow with the simple source terms by including these source terms in the lattice Boltzmann equation. Zhou (2011) overcomes the LBM for shallow water problem with the complex source terms by using the idea of centre scheme to manage the order of the accuracy of the source term. For the problem with the wetting – drying interface, the standard LBM does not work in calculation. However, some researches use the artificial assumptions such as a thin film and the extrapolation of unknow variable (Shafiai, 2013). Eventually, Liu & Zhou (2014) introduced the approach to solve this problem with wet – dry front by using the Taylor expansion and Chapman – Enskog procedure. In this work, we modify the Liu & Zhou (2014)'s idea for solving some model problem with wet – dry interface.

Method

1. Shallow water equations

In this section, the shallow water equations are introduced as the governing equation of the long wave phenomena. The shallow water equations with the source terms of bed slope and bed friction can be expressed as

$$\frac{\partial h}{\partial t}+\frac{\partial\left(hu_j\right)}{\partial x_j}=0, \tag{1}$$

$$\frac{\partial\left(hu_i\right)}{\partial t}+\frac{\partial\left(hu_iu_j\right)}{\partial x_j}=-\frac{g\partial h^2}{2\partial x_i}+\upsilon\frac{\partial^2\left(hu_i\right)}{\partial x_j\partial x_j}+F_i, \tag{2}$$

where i and j are indices computed based on the Einstein summation convention, h is the water depth, u_i are the velocity components in i directions, t is time, and $g \approx 9.81 \quad m/s^2$ is the gravitational acceleration. Here, υ is the kinematic viscosity definded by

$$\upsilon = C_s^2 \Delta t\left(\tau-\frac{1}{2}\right), \tag{3}$$

where τ is the relaxation time and C_s is the local sound speed.

The force term is described by

$$F_i = -gh\frac{\partial z_b}{\partial x_i} - \frac{\tau_{bi}}{\rho}, \tag{4}$$

where z_b is bed elevation, ρ is the fluid density, and τ_{bi} is the bed shear stress expressed as

$$\tau_{bi} = \rho C_b u_i \sqrt{u_j u_j}, \tag{5}$$

with the bed friction coefficient C_b.

2. Lattice Boltzmann method

In this study, 1D problem is considered the D1Q3 lattice shownd in Figure 1.

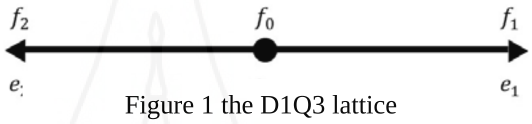

Figure 1 the D1Q3 lattice

The lattice Boltzmann equation with the distribution function, f_α, can be written as

$$f_\alpha\left(x+e_\alpha\Delta t, t+\Delta t\right) = f_\alpha\left(x,t\right) - \frac{1}{\tau}\left(f_\alpha - f_\alpha^{eq}\right) + w_\alpha \frac{\Delta t}{C_s^2} e_{\alpha i} F_i\left(x,t\right), \tag{6}$$

where e_α is the vector of the particle velocity at the α direction, $e_{\alpha i}$ is the component of e_α, w_α is the weighting factor constant (w_α =1/4 for D1Q3) , and f_α^{eq} is the equilibrium distribution function. For D1Q3,

$$f_\alpha^{eq} = \begin{cases} h - \dfrac{hu^2}{e^2} - \dfrac{gh^2}{2e^2}, & \alpha = 0, \\ \dfrac{gh^2}{4e^2} + \dfrac{hu^2}{2e^2} + \dfrac{hu}{2e}, & \alpha = 1, \\ \dfrac{gh^2}{4e^2} + \dfrac{hu^2}{2e^2} - \dfrac{hu}{2e}, & \alpha = 2. \end{cases} \tag{7}$$

where $e = \Delta x / \Delta t$: when Δx is the lattice step size, and Δt is the time step size.
By using the present distribution function, we acquire the unknown variables by the subsequent relations

$$h = \sum_\alpha f_\alpha, \quad u_i = \frac{1}{h}\sum_\alpha e_{\alpha i} f_\alpha. \tag{8}$$

The boundary conditions can be specified corresponding to the boundary of the problem (Zhou, 2004), except the problems with the wetting – drying front, which must be derived according to the wet – dry condition.

3. **The wet – dry boundary condition**

In this part, we follow the Liu & Zhou (2014)'s idea by using both mathematical concepts (the Taylor expansion and Chapman – Enskog procedure) for the non – equilibrium distribution function (Latt & Chopard, 2005), we obtain the following formula

$$f_\alpha = -\frac{gh\tau}{2e^2}\left(z_b\left(x+e_\alpha\Delta t\right)-z_b\left(x\right)\right)-\frac{\Delta t\tau}{2e^2}e_\alpha C_b u\left|u\right|-\tau\left(f_\alpha^{eq}\left(x+e_\alpha\Delta t\right)-f_\alpha^{eq}\left(x\right)\right). \tag{9}$$

This formula is used when f_α move from the dry lattice to the wet lattice, see Figure 2, where f_α is $f_2\left(d_1\right)$. For $f_0\left(d_1\right)$, it can be calculated by the average of f_0 at its neighboring lattice as

$$f_0\left(d_1\right)=\frac{f_0\left(d_2\right)+f_0\left(w_1\right)}{2}. \tag{10}$$

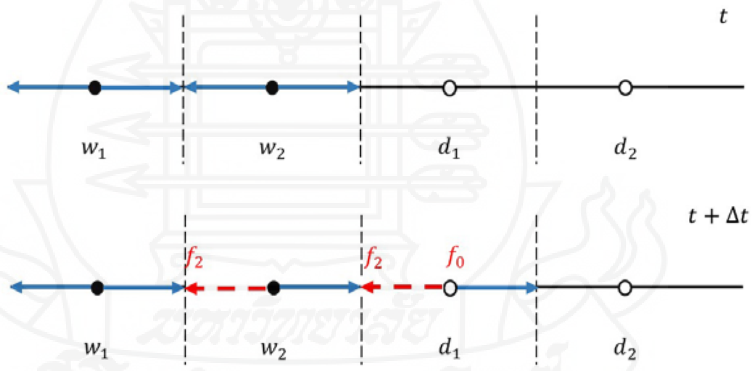

Figure 2 the wetting – drying interface.

4. Algorithm

The wet – dry interface based on an algorithm of Liu & Zhou (2014) is described as follow:

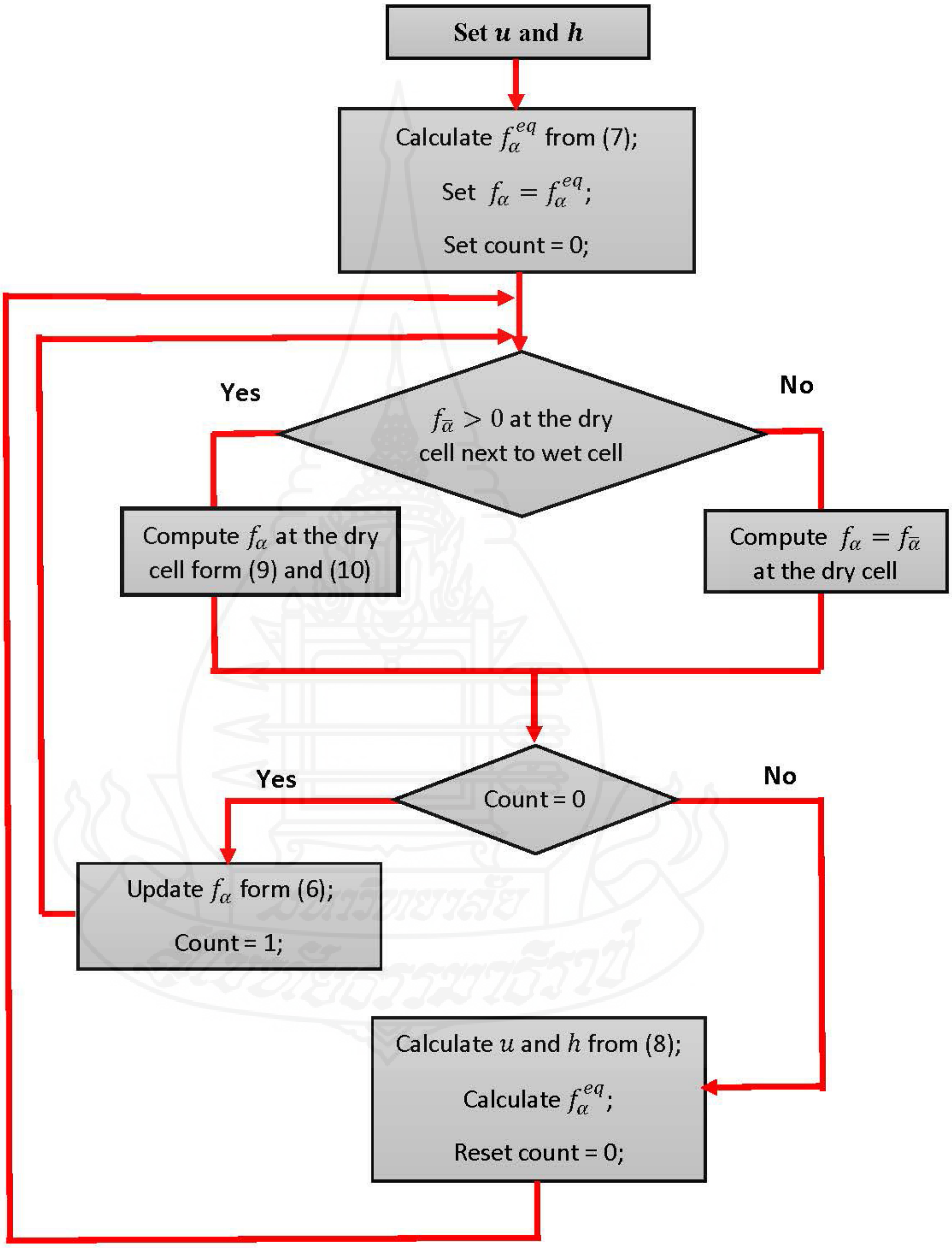

Figure 3 the algorithm for the wet – dry interface

Numerical results

A 1D wave run – down over a sloping bed

This example is studied in order to show the wet – dry interface when the bed topography emerses from the water surface. The channel is 500 m long with assigned 100 lattices. The bed topography is the variable of sloping bed expressed as

$$z_b(x) = 1.4 - 0.0028x$$

At the first step, the water level is set at 1.75 m hight as the initial condition. The solid boundary is specified at $x = 0$ m and the inlet boundary at $x = 500$ m where the water depth is governed by

$$h(500,t) = h_0 + \lambda \cos\left(2\pi \frac{t}{T}\right),$$

where $h_0 = 1$ m is the reference water surface, the amplitude if tidal wave $x = 0.75$ m, the tidal peried $T = 3600$ s, the Manning coefficient $n_b = 0.03$ and the relaxation time $\tau = 0.7$. The numerical results are shown in the following where the red line represents the bed topography and the blue line represents the water surface.

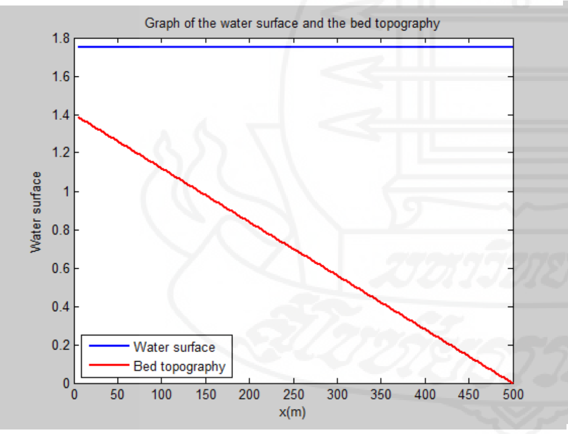

Figure 4.1 $t = 0.00s.$

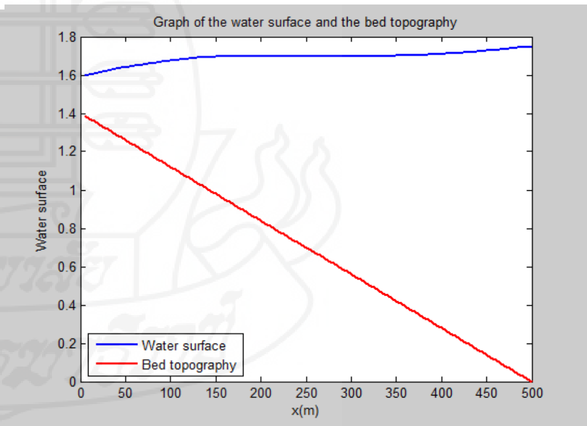

Figure 4.2 $t = 50.00s.$

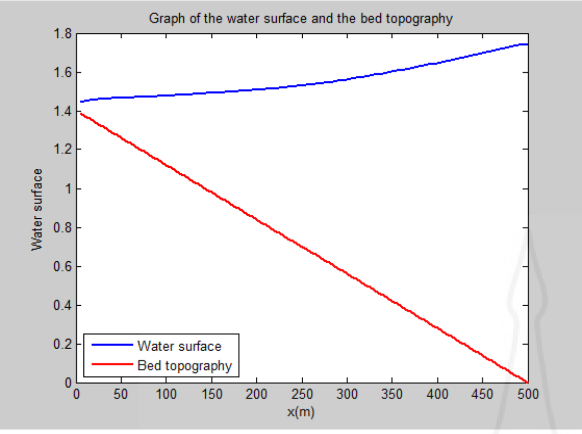

Figure 4.3 $t = 100.00s.$

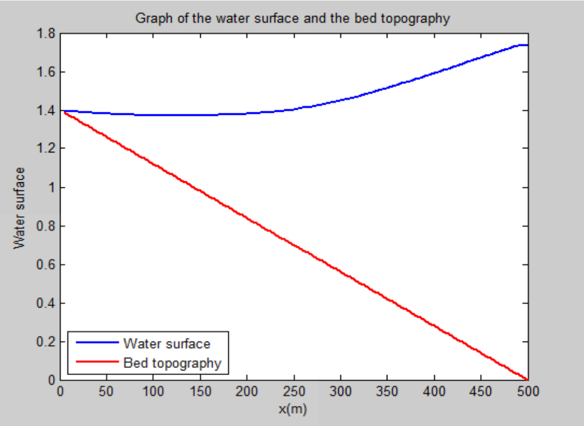

Figure 4.4 $t = 125.00s.$

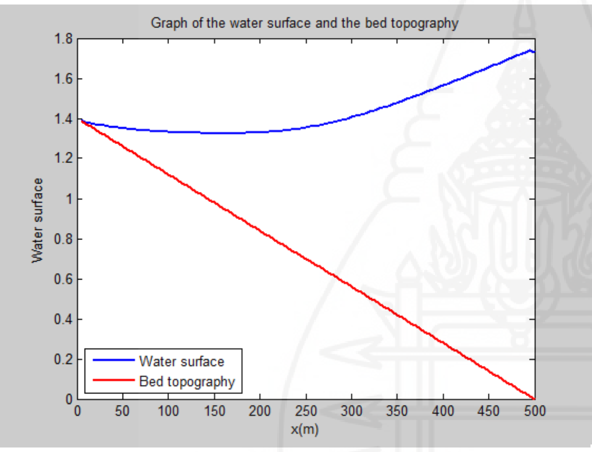

Figure 4.5 $t = 135.00s.$

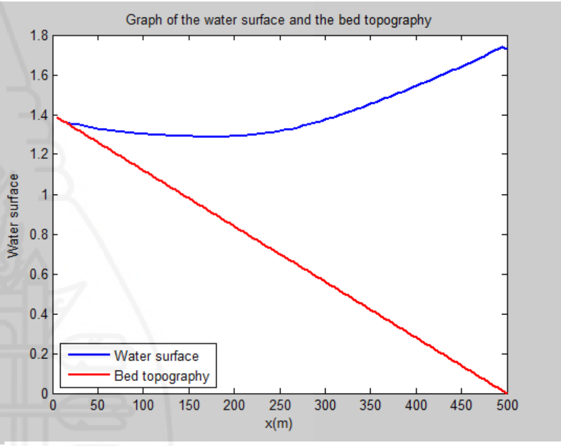

Figure 4.6 $t = 142.50s.$

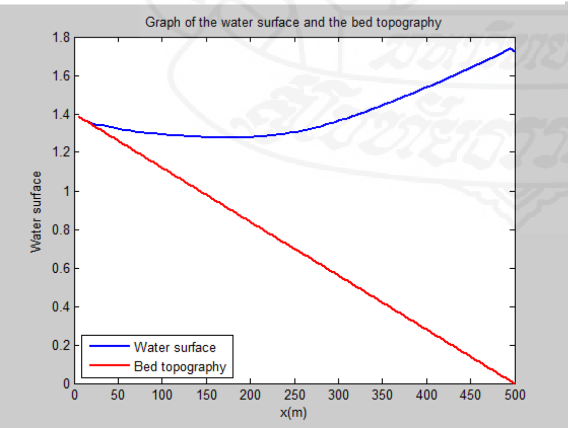

Figure 4.7 $t = 145.00s.$

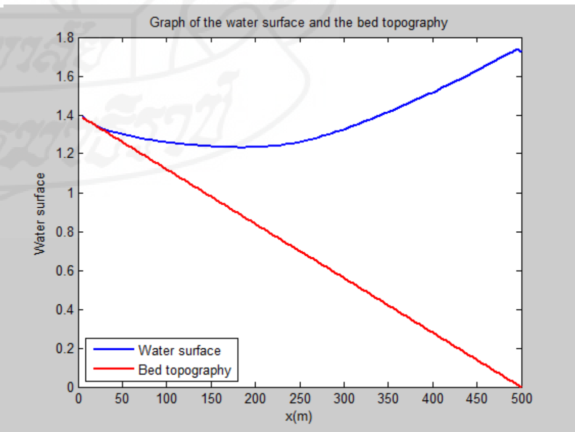

Figure 4.8 $t = 150.00s.$

Figure 4 Graph of the water surface and the bed topography.

From the above graphs, the figures 4.1, 4.2, 4.3 and 4.4 show that the weter surface (the blue line) is squeezing up to the bed topography (the red line). The figures 4.5, 4.6, 4.7 and 4.8 show that the bed topography emerses from the water surface, then the wet – dry interface occurs. Our new boundary condition is used. All of these results show that the wave are running down from the shoreline.

CONCLUSION

In this work, the lattice Boltzmann method with new boundary condition for the wet – dry interface can be applied to solve the shallow water flow problem when the wet – dry interface occurs under the variable of sloping bed. But if we use the complex bed topography instead of the variable of sloping bed. There are some mistakes which are the key point to research and develope for this problem.

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